Roll No. .... Total Pages : 04

BT-2/M-19 32026

APPLIED MATHEMATICS-II

AS-104N (Opt. (i))

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) If α, β, γ are the roots of the equation $2x^3+3x^2-x-1=0$, form the equation whose roots are $(1-\alpha)^{-1}, (1-\beta)^{-1}$ and $(1-\gamma)^{-1}$. 7.5

   (b) Show that the equation $x^4-10x^3+23x^2-6x-15=0$ can be transformed into reciprocal equation by diminishing the roots by 2. Hence solve the equation. 7.5

2. (a) State and prove the relation between beta and gamma function. 7.5

   (b) Using the method of differentiation under the integral sign, evaluate : 7.5

$$\int_0^{\pi/2} \log\left(\alpha^2\cos^2\theta+\beta^2\sin^2\theta\right)d\theta = \pi\log\frac{\alpha+\beta}{2}$$

Unit II

3. (a) Solve :

   (i) $L\left(e^{-4t}\int_0^t t\sin 3t\,dt\right)$

   (ii) $L^{-1}\left(\log\left(\frac{s+a}{s+b}\right)\right)$ 4+3.5

   (b) Only state the following properties of the Laplace Transform : 7.5

   (i) First shifting property
   (ii) Multiplication property
   (iii) Division property
   (iv) Derivative property
   (v) Integral property.

4. (a) State and prove the Convolution theorem and evaluate $L^{-1}\left(\frac{1}{s\left(1+s^2\right)}\right)$. 4+3.5

   (b) Using the Laplace transform, solve the differential equation $\frac{d^2y}{dt^2}+y=t\cos 2t$, given that :

$$y(0) = y'(0) = 0$$

Unit III

5. (a) Solve the differential equation : 7.5

$$\left(x^2y^2+xy+1\right)ydx+\left(x^2y^2-xy+1\right)xdy=0$$

(b) If the temperature of the body drops from 100°C to 60°C in one minute when the temperature of the surrounding is 20°C, what will be the temperature of the body at the end of the second minute ?

6. (a) Solve the differential equation : 7.5

$$\left(D^2+4\right)y=x^2\cos 2x$$

(b) Solve : 7.5

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}=2\sin\left[\log(1+x)\right]$$

Unit IV

7. (a) Find the directional derivative of $\phi=xy+yz+zx$ at the point P(1, 2, 0) in the direction of $\hat{i}+2\hat{j}+2\hat{k}$. 7.5

(b) Give the geometrical interpretation of divergence of a vector field. 7.5

8. (a) Evaluate by Green's theorem $\int_C\left(2x^2-y^2\right)dx+\left(x^2+y^2\right)dy$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2+y^2=a^2$. 7.5

(b) Using Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$, where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. 7.5